ITÜ FACULTY OF NAVAL ARCHITECTURE AND OCEAN ENGINEERING

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HYDRODYNAMIC ANALYSIS OF A RECTANGULAR FLOATING BREAKWATER IN REGULAR WAVES

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Contents

Introduction

- Motivation
  - Establish a simple mathematical basis for rectangular cylinder hydrodynamics
    - To develope a code for mooring problem of rectangular floating breakwaters
    - To have a mathematical tool for plunging rectangular cylinders intended to extract wave energy
- In this study
  - Why analytical method?
    - **Zheng, Y. H., Shen, Y. M., You, Y.G., Wu, B. J. Rong, L.** (2005). Hydrodynamic properties of two vertical truncated cylinders in waves. *Ocean Engineering*, ***32***, *241-271*
  - Why rectangular section?
    - **Hales, L. Z**., (1981). Floating Breakwaters: state of the art literature review. Technical Report no 81-1. US Army Engineer Waterways Experiment Station. Vicksburg, Mississippi, USA.
    - **Koftis, T., Prinos, P.,** On the hydrodynamic efficiency of floating breakwaters. Aristotle University of Thessaloniki, Greece.

Literature summary

- Kim (1969)
- Bai (1975)
- Sabuncu (1978)
- Sabuncu, Calışal (1981)

The neccesity of the study

- **This method**
  - **Makes the solution more simple**
  - **Needs shorter computing time**
  - **Converges very quickly**

- Sabuncu, T., Gören, Ö. (1981), (1983)
- Sabuncu, T., Çalışal, S. (1981), (1989)
- Pek, N. (1989)
- Gören, Ö., Söylemez, M. (2004)

For the solution;

- Separation of variables
- Matching technique

- Potential and linear theory
  - The flow is irrotational
  - Viscosity and surface tension neglected
  - Free surface condition and kinematic condition on the body are eliminated.
- The particle kinematics and the wave pressure are derived from the velocity potential.

Problem definition

- The problem will be divided into two independent situation:

Radiation Problem (Heave-Sway-Roll)

+

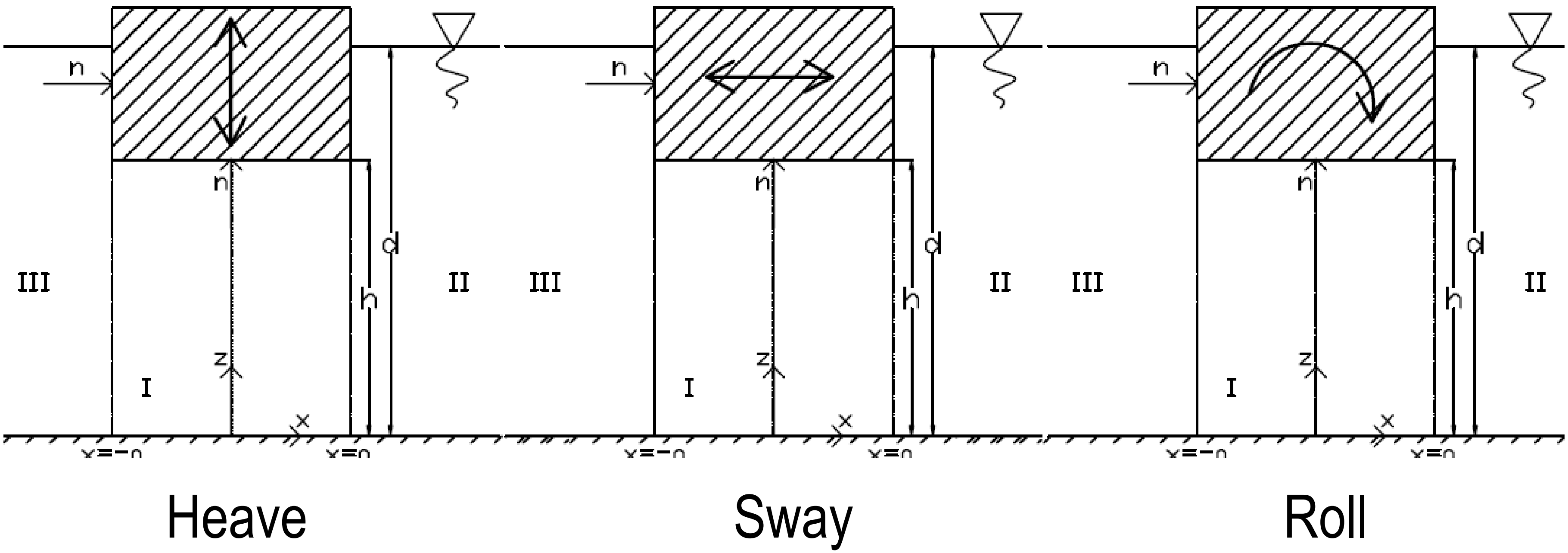
Diffraction Problem (Wave forces-moment)

=

Hydrodynamic analysis

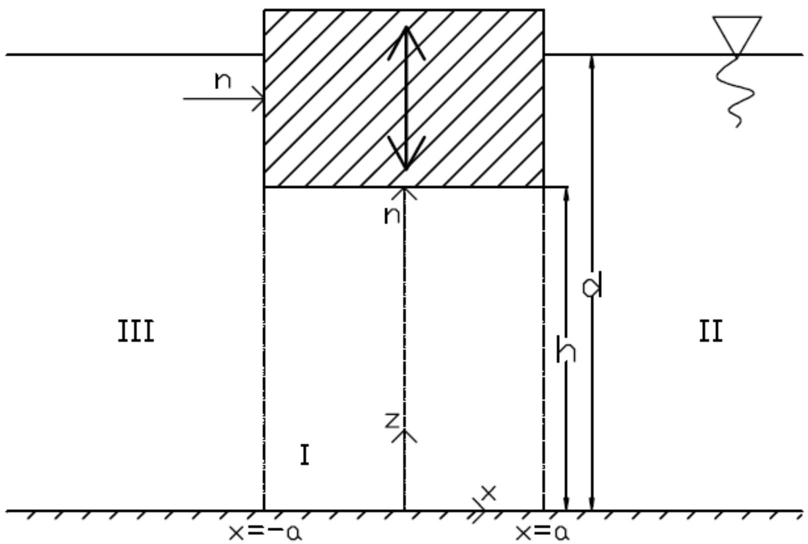
(Coupling effects are neglected)

Radiation problem



The boundary value problem(BVP)-Heave

$$\vec{V} = V_o e^{-i\omega t}\vec{k}$$



$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = 0$$

$$\Phi_{tt} + g\Phi_z = 0 \qquad (z = d)$$

$$\frac{\partial\Phi}{\partial z} = 0 \qquad (z = 0)$$

$$\frac{\partial\Phi}{\partial x} = 0 \quad (x = a, \;\; x = -a, \;\; h \le z \le d)$$

$$\frac{\partial\Phi}{\partial z} = V_0 \mathrm{e}^{-i\omega t} \qquad (-a < x < a \;\; \text{ve} \;\; z = h)$$

$$\Phi(x, z; t) = \mathrm{Re}\left\{V_0 d\phi(x, z)e^{-i\omega t}\right\}$$

The velocity potential is in dimensionless form $\phi(x, z)$ due to the $V_0 d$ term and will be used in this form.

The velocity potentials in region I, II & III

In region I

The potentials that are gained by using the separation of variables method

$$\varphi_I(x,z)=\frac{A_0}{2}+\sum_{n=1}^{\infty}A_n\frac{\cosh\frac{n\pi x}{h}}{\cosh\frac{n\pi a}{h}}\cos\frac{n\pi z}{h}+\frac{B_0}{2}\frac{x}{a}+\sum_{n=1}^{\infty}B_n\frac{\sinh\frac{n\pi x}{h}}{\sinh\frac{n\pi a}{h}}\cos\frac{n\pi z}{h}$$

$\varphi_I(x,z)$ expanded into Fourier series

$$\phi_I(x,z)=\phi_{I_h}(x,z)+\phi_{I_p}(x,z)$$

$$\phi_{I_p}(x,z)=\frac{1}{2hd}(z^2-x^2)$$ (Sabuncu, T.)

In region II & III

$\varphi_{II}(x,z)$ ve $\varphi_{III}(x,z)$ expanded into Eigen functions.

$$\varphi_{II}(x,z) = C_0 \frac{e^{ik_0 x}}{e^{ik_0 a}} Z_0(z) + \sum_{m=1}^{\infty} C_m \frac{e^{-k_m x}}{e^{-k_m a}} Z_m(z)$$

$$\varphi_{III}(x,z) = D_0 \frac{e^{-ik_0 x}}{e^{ik_0 a}} Z_0(z) + \sum_{m=1}^{\infty} D_m \frac{e^{k_m x}}{e^{-k_m a}} Z_m(z)$$

Matching conditions

Pressure

$$\varphi_I(x,z)=\varphi_{II}(x,z) \qquad x=a\ ,\ 0\le z\le h$$

$$\varphi_I(x,z)=\varphi_{III}(x,z) \qquad x=-a\ ,\ 0\le z\le h$$

Velocity

$$\frac{\partial\varphi_I(x,z)}{\partial x}=\frac{\partial\varphi_{II}(x,z)}{\partial x} \qquad (x=a\ ,\ 0\le \mathrm{z}\le \mathrm{d})$$

$$\frac{\partial\varphi_I(x,z)}{\partial x}=\frac{\partial\varphi_{III}(x,z)}{\partial x} \qquad (x=-a\ ,\ 0\le \mathrm{z}\le \mathrm{d})$$

1st Equation system/set

$$A_n + B_n = \frac{2}{h}\int_0^h \varphi_I(a,z)\cos\frac{n\pi z}{h}dz$$

$$A_n + B_n = \frac{2}{h}\int_0^h C_0 Z_0(z)\cos\frac{n\pi z}{h}dz + \frac{2}{h}\int_0^h \sum_{m=1}^{\infty} C_m Z_m(z)\cos\frac{n\pi z}{h}dz$$

$$A_0 + B_0 - \sum_{m=0}^{\infty} C_m L_{m0} = -\frac{h}{3d} + \frac{a^2}{hd} \qquad n = 0$$

$$A_n + B_n - \sum_{m=0}^{\infty} C_m L_{mn} = -\frac{2h}{d}\frac{(-1)^n}{(n\pi)^2} \qquad n \geq 1, 2...$$

2nd Equation system

$$A_n - B_n = \frac{2}{h}\int_0^h \varphi_I(-a,z)\cos\frac{n\pi z}{h}dz$$

$$A_n - B_n = \frac{2}{h}\int_0^h D_0 Z_0(z)\cos\frac{n\pi z}{h}dz + \frac{2}{h}\int_0^h \sum_{m=1}^{\infty} D_m Z_m(z)\cos\frac{n\pi z}{h}dz$$

$$A_0 - B_0 - \sum_{m=0}^{\infty} D_m L_{m0} = -\frac{h}{3d} + \frac{a^2}{hd} \qquad n = 0$$

$$A_n - B_n - \sum_{m=0}^{\infty} D_m L_{mn} = -\frac{2h}{d}\frac{(-1)^n}{(n\pi)^2} \qquad n \geq 1, 2...$$

3rd Equation system

$$ik_0 dC_0 = \int_0^h \frac{\partial \phi_{II}(a,z)}{\partial x} Z_0(z)dz + \int_h^d \frac{\partial \phi_{II}(a,z)}{\partial x} Z_0(z)dz \quad \text{m=0}$$

$$-k_m dC_m = \int_0^h \frac{\partial \phi_{II}(a,z)}{\partial x} Z_m(z)dz + \int_h^d \frac{\partial \phi_{II}(a,z)}{\partial x} Z_m(z)dz \quad \text{m} \geq 1$$

$$\sum_{n=1}^{\infty} A_n \frac{n\pi}{2} \tanh\frac{n\pi a}{h} L_{0n} + B_0 \frac{h}{4a} L_{00} + \sum_{n=1}^{\infty} B_n \frac{n\pi}{2} \coth\frac{n\pi a}{h} L_{0n} - C_0 ik_0 d = \frac{a}{h} \frac{N_0^{-1/2}}{k_0 d} \sinh k_0 h$$

$$\sum_{n=1}^{\infty} A_n \frac{n\pi}{2} \tanh\frac{n\pi a}{h} L_{mn} + B_0 \frac{h}{4a} L_{m0} + \sum_{n=1}^{\infty} B_n \frac{n\pi}{2} \coth\frac{n\pi a}{h} L_{mn} + \sum_{m=1}^{\infty} C_m k_m d = \frac{a}{h} \frac{N_m^{-1/2}}{k_m d} \sin k_m h$$

4th Equation system

$$-ik_0 dD_0 = \int_0^h \frac{\partial \phi_{III}(-a,z)}{\partial x} Z_0(z)dz + \int_h^d \frac{\partial \phi_{III}(-a,z)}{\partial x} Z_0(z)dz \qquad \text{m=0}$$

$$k_m dD_m = \int_0^h \frac{\partial \phi_{III}(-a,z)}{\partial x} Z_m(z)dz + \int_h^d \frac{\partial \phi_{III}(-a,z)}{\partial x} Z_m(z)dz \qquad m \geq 1$$

$$-\sum_{n=1}^{\infty} A_n \frac{n\pi}{2} \tanh \frac{n\pi a}{h} L_{0n} + B_0 \frac{h}{4a} L_{00} + \sum_{n=1}^{\infty} B_n \frac{n\pi}{2} \coth \frac{n\pi a}{h} L_{0n} - D_0 ik_0 d = \frac{a}{h} \frac{N_0^{-1/2}}{k_0 d} \sinh k_0 h$$

$$-\sum_{n=1}^{\infty} A_n \frac{n\pi}{2} \tanh \frac{n\pi a}{h} L_{mn} + B_0 \frac{h}{4a} L_{m0} + \sum_{n=1}^{\infty} B_n \frac{n\pi}{2} \coth \frac{n\pi a}{h} L_{mn} + \sum_{m=1}^{\infty} D_m k_m d = \frac{a}{h} \frac{N_m^{-1/2}}{k_m d} \sin k_m h$$

Complete set of linear equations

$$A_0 + B_0 - \sum_{m=0}^{\infty} C_m L_{m0} = -\frac{h}{3d} + \frac{a^2}{hd} \qquad n = 0$$

$$A_n + B_n - \sum_{m=0}^{\infty} C_m L_{mn} = -\frac{2h}{d}\frac{(-1)^n}{(n\pi)^2} \qquad n \geq 1, 2...$$

$$A_0 - B_0 - \sum_{m=0}^{\infty} D_m L_{m0} = -\frac{h}{3d} + \frac{a^2}{hd} \qquad n = 0$$

$$A_n - B_n - \sum_{m=0}^{\infty} D_m L_{mn} = -\frac{2h}{d}\frac{(-1)^n}{(n\pi)^2} \qquad n \geq 1, 2...$$

Linear equation system is solved with a Fortran code which uses Gauss elimination method.

$$\sum_{n=1}^{\infty} A_n \frac{n\pi}{2} \tanh\frac{n\pi a}{h} L_{0n} + B_0 \frac{h}{4a} L_{00} + \sum_{n=1}^{\infty} B_n \frac{n\pi}{2} \coth\frac{n\pi a}{h} L_{0n} - C_0 i k_0 d = \frac{a}{h}\frac{N_0^{-1/2}}{k_0 d} \sinh k_0 h$$

$$\sum_{n=1}^{\infty} A_n \frac{n\pi}{2} \tanh\frac{n\pi a}{h} L_{mn} + B_0 \frac{h}{4a} L_{m0} + \sum_{n=1}^{\infty} B_n \frac{n\pi}{2} \coth\frac{n\pi a}{h} L_{mn} + \sum_{m=1}^{\infty} C_m k_m d = \frac{a}{h}\frac{N_m^{-1/2}}{k_m d} \sin k_m h$$

$$-\sum_{n=1}^{\infty} A_n \frac{n\pi}{2} \tanh\frac{n\pi a}{h} L_{0n} + B_0 \frac{h}{4a} L_{00} + \sum_{n=1}^{\infty} B_n \frac{n\pi}{2} \coth\frac{n\pi a}{h} L_{0n} - D_0 i k_0 d = \frac{a}{h}\frac{N_0^{-1/2}}{k_0 d} \sinh k_0 h$$

$$-\sum_{n=1}^{\infty} A_n \frac{n\pi}{2} \tanh\frac{n\pi a}{h} L_{mn} + B_0 \frac{h}{4a} L_{m0} + \sum_{n=1}^{\infty} B_n \frac{n\pi}{2} \coth\frac{n\pi a}{h} L_{mn} + \sum_{m=1}^{\infty} D_m k_m d = \frac{a}{h}\frac{N_m^{-1/2}}{k_m d} \sin k_m h$$

During the solution steps

$$\frac{1}{h}\int_0^h Z_\upsilon(z) Z_\mu(z)\,dz = \delta_{\upsilon\mu}$$

$$Z_0 = N_0^{-1/2}\cosh(k_0 z) \qquad \text{(m=0)}$$

$$Z_m = N_m^{-1/2}\cos(k_m z) \qquad (\text{m} \geq 1)$$

$$N_0 = \frac{1}{2}\left\{1 + \frac{\sinh(2k_0 d)}{2k_0 d}\right\} \qquad\qquad N_m = \frac{1}{2}\left\{1 + \frac{\sin(2k_m d)}{2k_m d}\right\}$$

$$L_{0n} = \frac{2N_0^{-1/2}(-1)^n k_0 h \sinh(k_0 h)}{(n\pi)^2 + (k_0 h)^2} \qquad (m = 0 \text{ ve } n = 0,1,2...)$$

$$L_{mn} = \frac{2N_m^{-1/2}(-1)^n k_m h \sin(k_m h)}{(k_m h)^2 - (n\pi)^2} \qquad (m \geq 1 \text{ ve } n = 0,1,2...)$$

Added mass and damping coefficients

- By integrating the pressure

$$\vec{F} = -\rho \frac{\partial}{\partial t} \int_C \Phi(a, z; t) \vec{n} ds$$

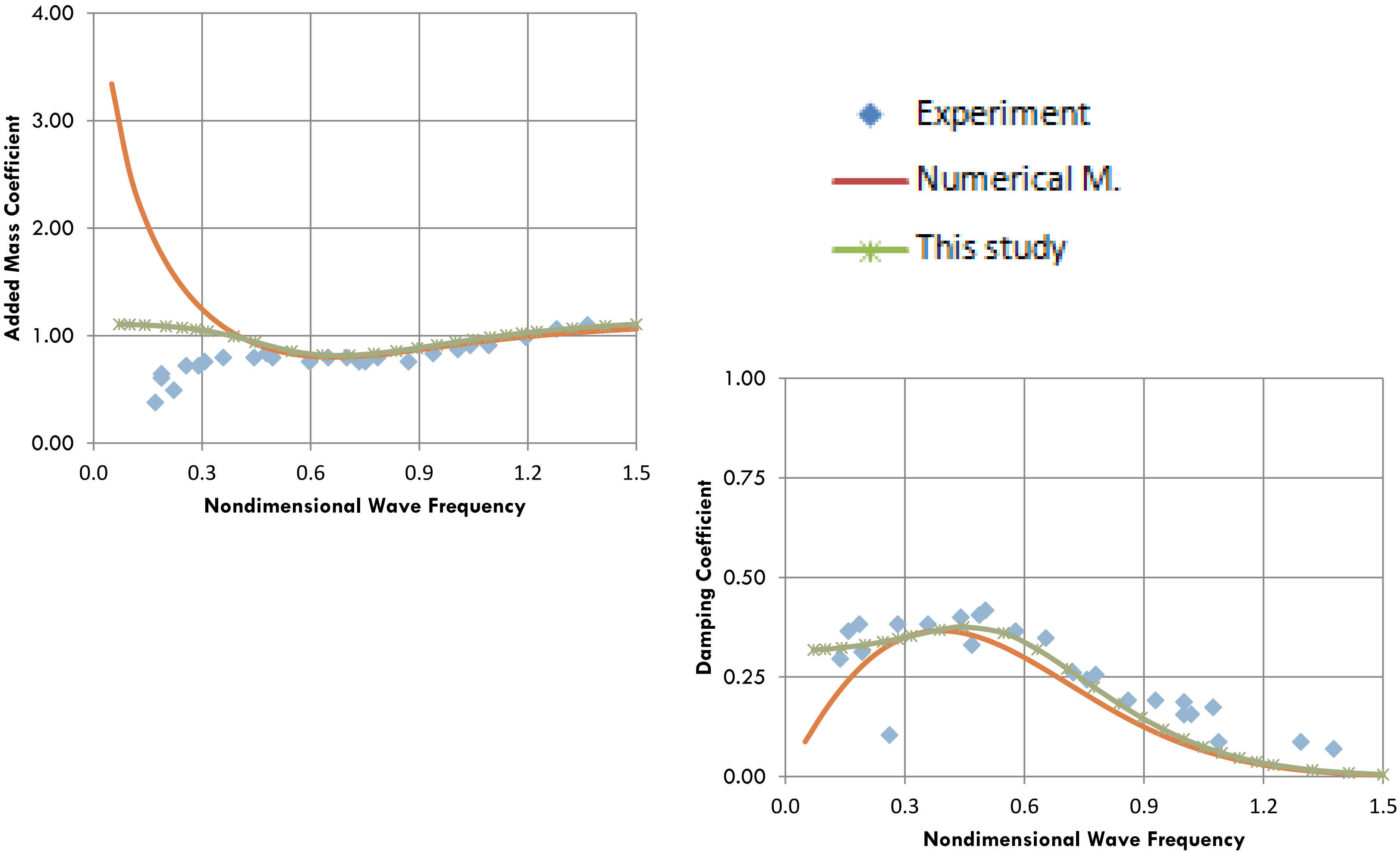
$$p = -\rho \frac{\partial \Phi(x, z; t)}{\partial t}$$

$$a_V \dot{V} + b_V V + \int_{-a}^{a} P dx = 0$$

$$a_V \dot{V} + b_V V + F_V = 0$$

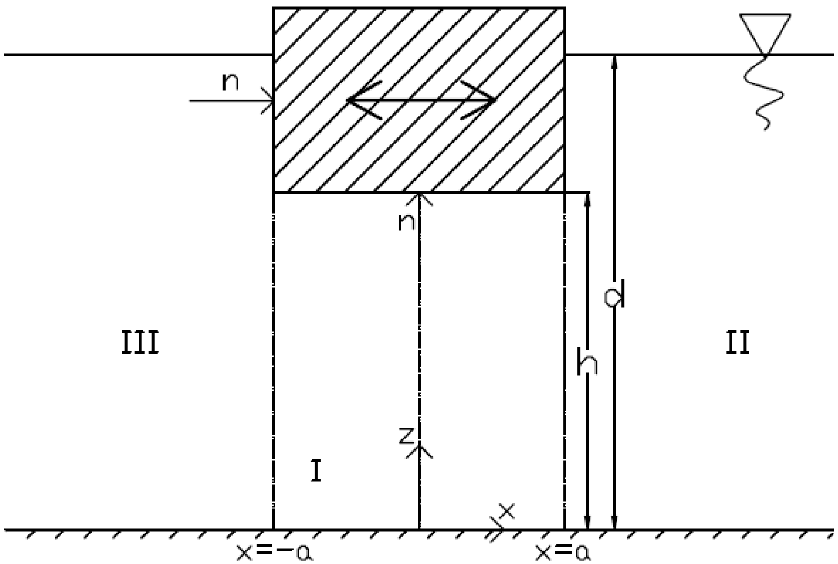
$$\frac{a_z}{\rho a^2} + i \frac{b_z}{\rho a^2 \omega} = \frac{h}{a} - \frac{a}{3h} + \frac{d}{a} A_0 + \frac{2d}{a} \sum_{n=1}^{\infty} A_n (-1)^n \left(\frac{h}{n\pi a} \right) \tanh\left(\frac{h}{n\pi a} \right)$$

Hydrodynamic coefficients results and comparison



BVP for sway motion

$$\vec{V} = V_o e^{-i\omega t}\vec{i}$$



$$\Phi(x,z;t) = \mathrm{Re}\left\{V_0 d\phi(x,z)e^{-i\omega t}\right\}$$

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = 0$$

$$\Phi_{tt} + g\Phi_z = 0 \qquad (z = d)$$

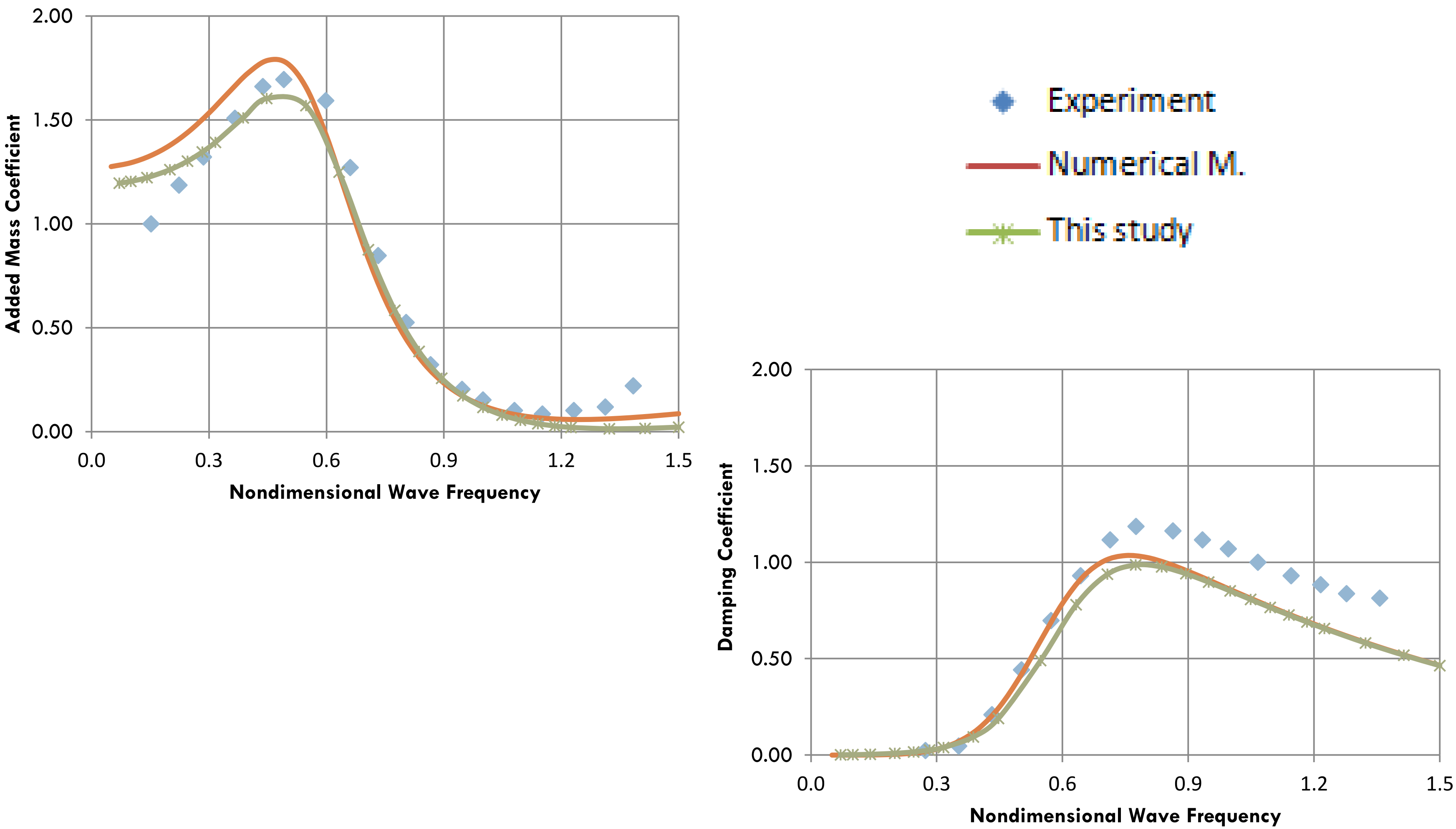
$$\frac{\partial\Phi}{\partial z} = 0 \qquad (z = 0)$$

$$\frac{\partial\Phi}{\partial x} = V_o e^{-i\omega t} \quad (x = a,\ \ x = -a,\ \ h \le z \le d)$$

$$\frac{\partial\Phi}{\partial z} = 0 \quad (-a < x < a \ \ \text{ve} \ \ z = h)$$

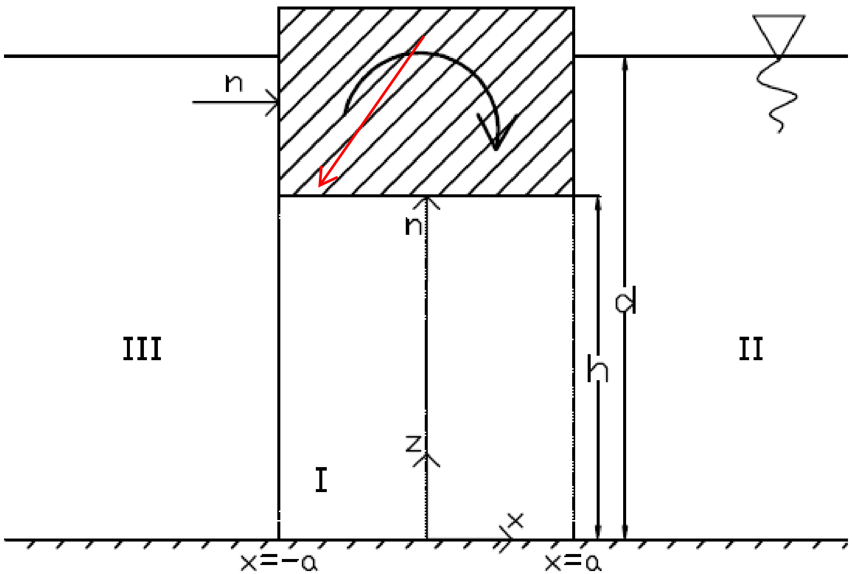
The velocity potential is in dimensionless form $\phi(x,z)$ due to the $V_0 d$ term and will be used in this form.

Hydrodynamic coefficients results and comparison



BVP for roll motion

$$\vec{\Omega} = \Omega_o e^{-i\omega t} \vec{j}$$



$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$

$$\Phi_{tt} + g\Phi_z = 0 \qquad (z = d)$$

$$\frac{\partial \Phi}{\partial z} = 0 \qquad (z = 0)$$

$$\frac{\partial \Phi}{\partial x} = \vec{\Omega}(z - d) \quad (x = a, \;\; x = -a, \;\; h \le z \le d)$$

$$\frac{\partial \Phi}{\partial z} = -\vec{\Omega} x \quad (-a < x < a \;\; \text{ve} \;\; z = h)$$

$$\Phi(x, z; t) = \mathrm{Re}\left\{ \Omega_0 d^2 \phi(x, z) e^{-i\omega t} \right\}$$

The velocity potential is in dimensionless form $\phi(x, z)$ due to the $\Omega_0 d^2$ term and will be used in this form.

For roll motion

- Particular solution (presently introduced)

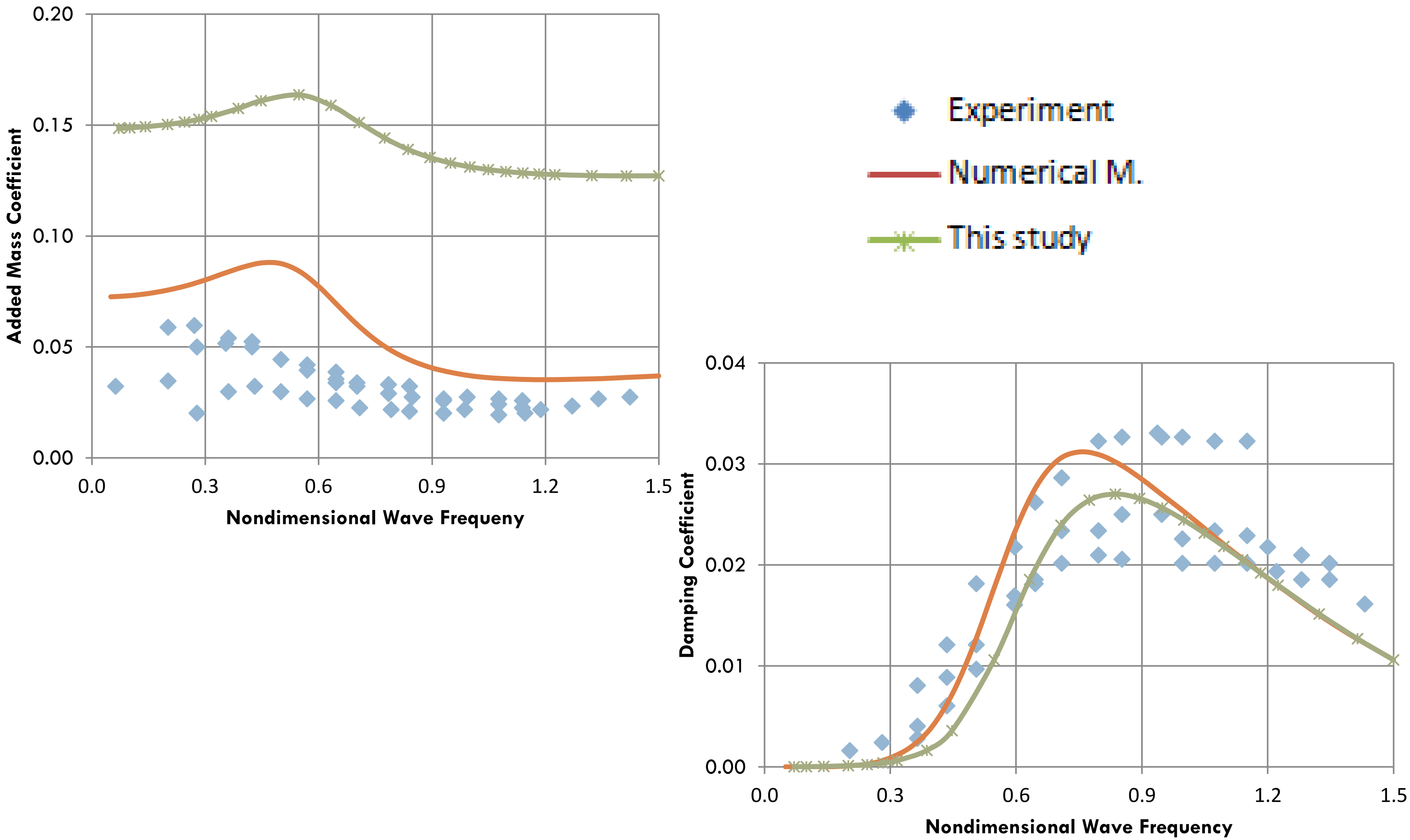
$$\phi_{I_p}(x,z) = \frac{x^3}{6hd^2} - \frac{xz^2}{2hd^2}$$

- For hydrodynamic coefficients

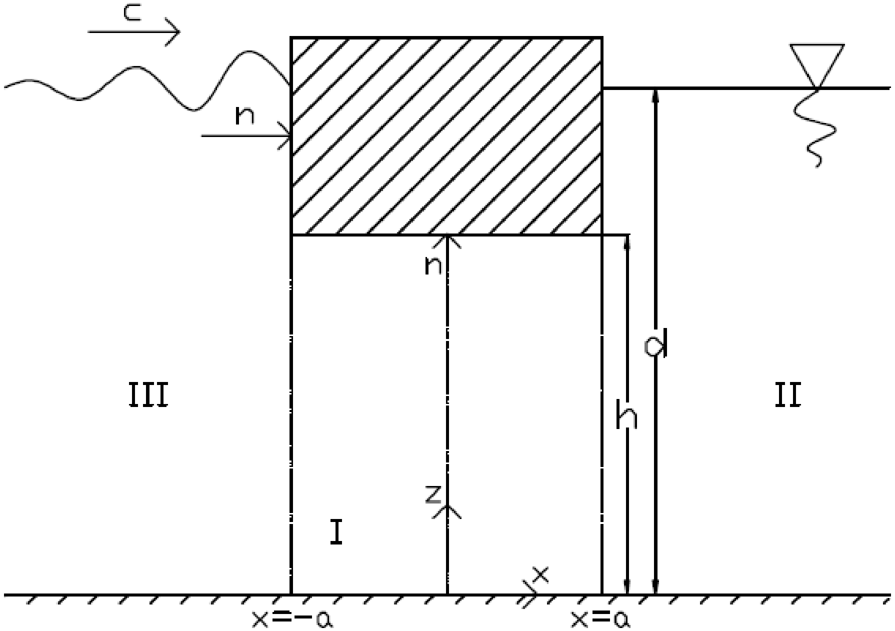
$$\vec{M} = \iint_S p(\vec{r} \times \vec{n}) ds$$

$$\vec{r} = x\vec{i} + (z-d)\vec{k}$$

Hydrodynamic coefficients results and comparison



Wave force and moment (Diffraction) analysis



$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial z^2}=0$$

$$\Phi_{tt}+g\Phi_z=0 \qquad (z=d)$$

$$\frac{\partial\Phi}{\partial z}=0 \qquad (z=0)$$

$$\frac{\partial\Phi}{\partial n}=0$$

$$\Phi(x,z;t)=\mathrm{Re}\left\{\zeta_i c\phi(x,z)e^{-i\omega t}\right\}$$

The velocity potential is in dimensionless form $\phi(x,z)$ due to the $\zeta_i c$ term and will be used in this form.

Velocity potentials in regions

$$\phi_I(x,z) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \frac{\cosh\frac{n\pi x}{h}}{\cosh\frac{n\pi a}{h}} \cos\frac{n\pi z}{h} + \frac{B_0}{2}\frac{x}{a} + \sum_{n=1}^{\infty} B_n \frac{\sinh\frac{n\pi x}{h}}{\sinh\frac{n\pi a}{h}} \cos\frac{n\pi z}{h}$$

$$\phi_{II}(x,z) = C_0 \frac{e^{ik_0 x}}{e^{ik_0 a}} Z_0(z) + \sum_{m=1}^{\infty} C_m \frac{e^{-k_m x}}{e^{-k_m a}} Z_m(z)$$

$$\phi_{III}(x,z) = D_0 \frac{e^{-ik_0 x}}{e^{ik_0 a}} Z_0(z) + \sum_{m=1}^{\infty} D_m \frac{e^{k_m x}}{e^{-k_m a}} Z_m(z) - \frac{\cosh(k_0 z)}{\sinh(k_0 d)} e^{ik_0 x}$$

Wave force and moment

$$\vec{F} = \iint_S p\vec{n}ds \qquad \vec{M} = \iint_S p(\vec{r} \times \vec{n})ds$$

$$p = -\rho \frac{\partial \Phi}{\partial t}$$

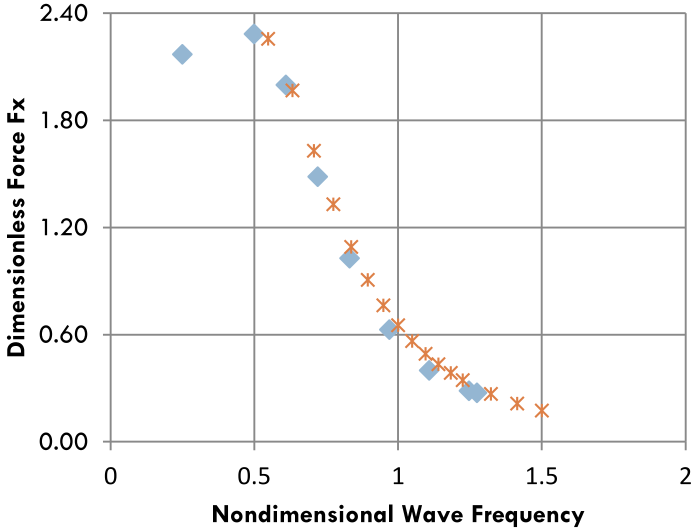
$$X = -\rho \int_h^d \frac{\partial \Phi_{III}(-a,z)}{\partial t} dz + \rho \int_h^d \frac{\partial \Phi_{II}(a,z)}{\partial t} dz$$

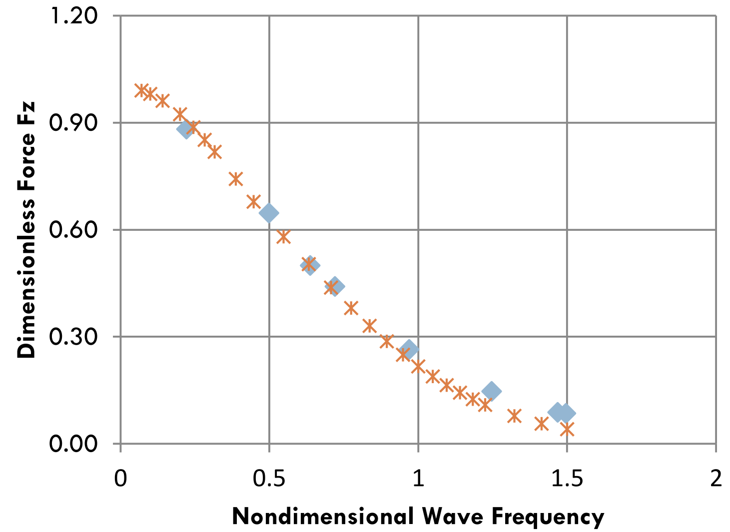
$$Z = -\rho \int_{-a}^{a} \frac{\partial \Phi_I(x,h)}{\partial t} dz$$

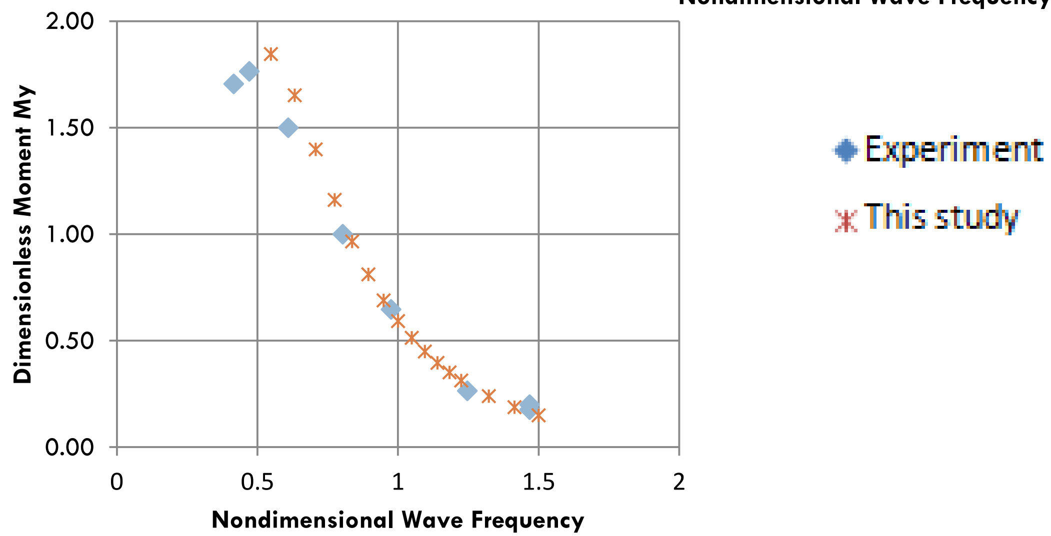
$$M = -\rho \int_h^d \frac{\partial \Phi_{III}(-a,z)}{\partial t} (z-d)dz + \rho \int_{-a}^{a} \frac{\partial \Phi_I(x,h)}{\partial t} xdz + \rho \int_h^d \frac{\partial \Phi_{II}(a,z)}{\partial t} (z-d)dz$$

Söylemez, M., Gören, Ö., (2003). Diffraction of oblique waves by thick rectangular barriers. *Applied Ocean Research*, **25**, 345-353.

Wave force and moment results and comparison







Equation of motion

$$\begin{pmatrix} \vec{F} \\ \vec{M} \end{pmatrix} = -\rho g \iint_{S_B} \begin{pmatrix} \vec{n} \\ \vec{r} x \vec{n} \end{pmatrix} z ds - \rho \operatorname{Re} \sum_{j=1}^{6} i\omega \, e^{i\omega t} \iint_{S_B} \begin{pmatrix} \vec{n} \\ \vec{r} x \vec{n} \end{pmatrix} \phi_j ds - \rho \operatorname{Re} i\omega \, e^{i\omega t} \iint_{S_B} \begin{pmatrix} \vec{n} \\ \vec{r} x \vec{n} \end{pmatrix} (\phi_0 + \phi_7) ds$$

- Radiation problem analysis
  - Added mass (acceleration)
  - Damping (velocity)
- Diffraction problem analysis
  - Wave load and moment
- Hydrostatic restoring forces and moment
  - Force and moment (displacement)

Equation of motion

$$(m_{11}+a_{11})\bar{\xi}_1(-\omega^2)e^{-i\omega t}+b_{11}\bar{\xi}_1(-i\omega)e^{-i\omega t}=\bar{F}_x e^{-i\omega t}$$

$$\frac{\bar{X}}{\zeta_a}=\frac{F_x\rho g a}{(m+a_{ij})(-\omega^2)+b_{ij}(-i\omega)}$$

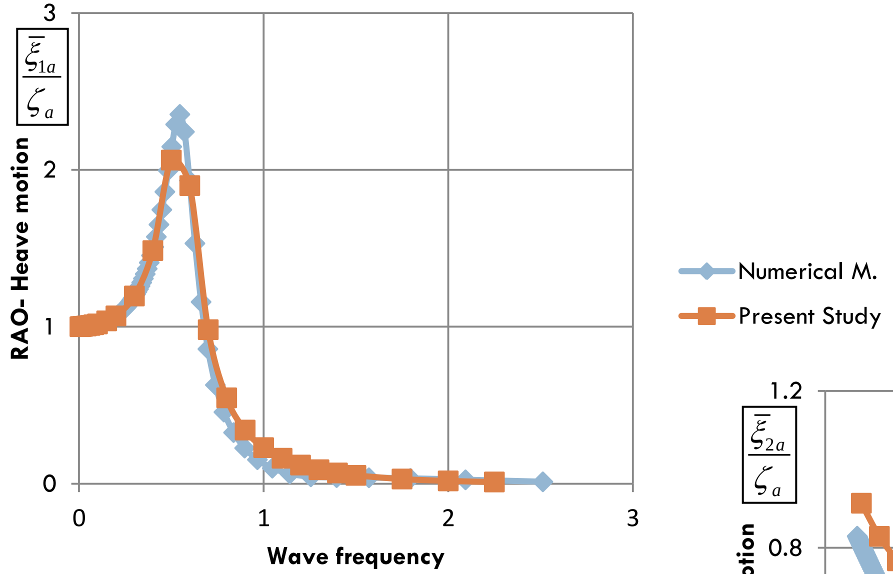
$$(m_{22}+a_{22})\bar{\xi}_2(-\omega^2)e^{-i\omega t}+b_{22}\bar{\xi}_2(-i\omega)e^{-i\omega t}+c_{22}\bar{\xi}_2 e^{-i\omega t}=\bar{F}_z e^{-i\omega t}$$

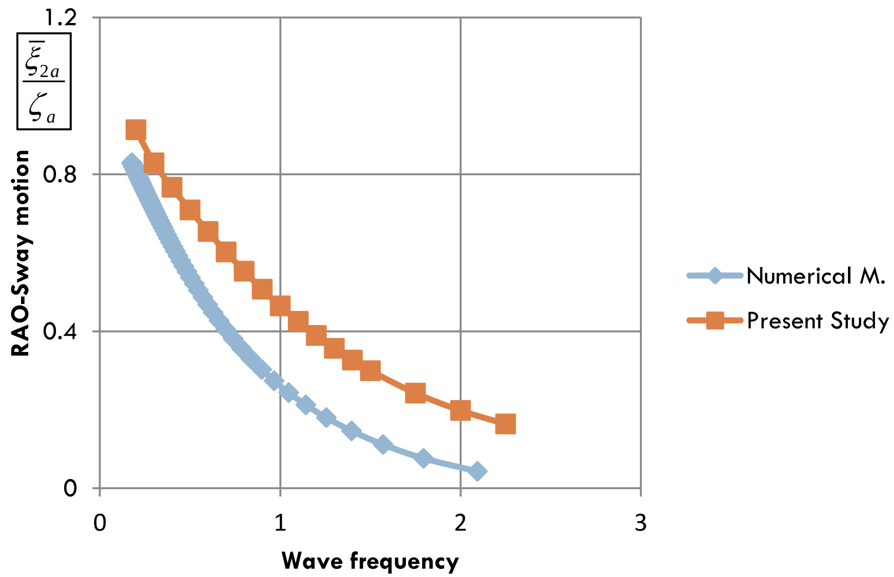
$$\frac{\bar{Z}}{\zeta_a}=\frac{F_z\rho g a}{(m+a_{ij})(-\omega^2)+b_{ij}(-i\omega)+c_{ij}}$$

$$(m_{33}+a_{33})\bar{\xi}_3(-\omega^2)e^{-i\omega t}+b_{33}\bar{\xi}_3(-i\omega)e^{-i\omega t}+c_{33}\bar{\xi}_3 e^{-i\omega t}=\bar{M}_y e^{-i\omega t}$$

$$\frac{\bar{\xi}_{3a}}{\alpha_m}=\frac{\bar{M}_y\rho g a^2}{(I_{33}+a_{33})(-\omega^2)+b_{33}(-i\omega)+c_{33}}$$

Response amplitude operator (RAO)





Results & Discussion

- For the radiation problem
  - Good agreement with the experiment results
  - For low frequency values the comparison with the experimental work is much more better than the numerical methods.
- Also wave force and moment values presented are in good agreement with the experimental work.
- Calculation method and getting the result in short time are the advantages when compared to numerical methods.

Field of application



Not only for floating breakwaters

- A design approach for new landscape requirements
  - for social activities, entertainment establishments or living
  - for industrial factories or military facilities
  - Terminals (such as for container ships, airports etc)
- Seakeeping characteristics of rectangular structures (Strip theory)
- Calculating the wave force and moment of wavemakers

Future work

- Planned future works as a continuation of the present study are
  - Generating a code for hydrodynamic analysis of moored floating structures by the linear theory approach
  - Nonlinear analysis of the same problem
  - Wave energy conversion in the shoreline by considering rectangular cylinder as a source of driving unit on linear generators.

Thank you for your attention