Design of Harmonic Filters for Renewable Energy Applications

Master Thesis Report
By:

Bhunesh Kumar

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Department of Wind Energy
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Supervisor: Dr. Bahri Uzunoglu
Wind Energy Dept. HGO Sweden

Examiner: Dr. Stefan Ivanell
Wind Energy Dept. HGO Sweden
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Abstract

Harmonics are created by non-linear devices connected to the power system. Power system harmonics are multiples of the fundamental power system frequency and these harmonic frequencies can create distorted voltages and currents. Distortion of voltages and currents can affect the power system adversely causing power quality problems. Therefore, estimation of harmonics is of high importance for efficiency of the power system network. The problem of harmonic loss evaluation is of growing importance for renewable power system industry by impacting the operating costs and the useful life of the system components.

Non-linear devices such as power electronics converters can inject harmonics alternating currents (AC) in the electrical power system. The number of sensitive loads that require ideal sinusoidal supply voltage for their proper operation has been increasing. To maintain the quality limits proposed by standards to protect the sensitive loads, it is necessary to include some form of filtering device to the power system. Harmonics also increases overall reactive power demanded by equivalent load. Filters have been devised to achieve an optimal control strategy for harmonic alleviation problems.

To achieve an acceptable distortion, increase the power quality and to reduce the harmonics hence several three phase filter banks are used and connected in parallel. In this thesis, high order harmonics cases have been suppressed by employing variants of Butterworth, Chebyshev and Cauer filters. MATLAB/SIMULINK wind farm model was used to generate and analyze the different harmonics magnitude and frequency. High voltage direct current (HVDC) lines for an electrical grid that is more than 50km far away wind farm generation plant was investigated for harmonics. These HVDC lines are also used in offshore wind farm plant. Investigated three-phase harmonics filters are shunt elements that are used in power systems for decreasing voltage distortion and for correcting the power factor.

Renewable energy sources are not the stable source of energy generation like wind, solar and tidal e.t.c. Though they are secondary sources of generation and hard to connect with electrical grid. In near future the technique is to use the wave digital filter (WDF) or circulator-tree wave digital filter (CTWDF) for the renewable energy application can be employed to mitigate the harmonics. These WDF and CTWDF can be used in HVDC lines and smart grid applications. A preliminary analysis is conducted for such a study.

Keywords: Digital Filter, Impulse Response, Wave Digital Filter, Circulator-Tree WAVE Digital Filter, Harmonics Filter, Renewable energy Filter, Wind Energy Filter, Butterworth, Chebyshev and Cauer filters, Low-pass, High-pass and Band-pass filters
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### Acronyms

<table>
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<tr>
<th>RE</th>
<th>Renewable energy</th>
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<tbody>
<tr>
<td>AC</td>
<td>Alternative Current</td>
</tr>
<tr>
<td>HVDC</td>
<td>High voltage direct current</td>
</tr>
<tr>
<td>WDF</td>
<td>Wave digital Filter</td>
</tr>
<tr>
<td>CTEDF</td>
<td>Circulator-tree wave digital filter</td>
</tr>
<tr>
<td>PQ</td>
<td>Power Quality</td>
</tr>
<tr>
<td>ASDs</td>
<td>Adjustable-speed motor drives</td>
</tr>
<tr>
<td>DC</td>
<td>Direct current</td>
</tr>
<tr>
<td>APF</td>
<td>Active power filter</td>
</tr>
<tr>
<td>IGBT</td>
<td>Insulated gate bipolar transistor</td>
</tr>
<tr>
<td>HPF</td>
<td>High pass filter</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital signal processor</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete fourier transform</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast fourier Transform</td>
</tr>
<tr>
<td>THD</td>
<td>Total harmonic distortion</td>
</tr>
<tr>
<td>RMS</td>
<td>Root mean square</td>
</tr>
<tr>
<td>EEG</td>
<td>Electroencephalograms</td>
</tr>
<tr>
<td>EKG</td>
<td>Electrocardiograph</td>
</tr>
<tr>
<td>FM</td>
<td>Frequency modulation</td>
</tr>
<tr>
<td>LTI</td>
<td>Linear time Invariant</td>
</tr>
<tr>
<td>SCRs</td>
<td>Silicon-controlled rectifiers</td>
</tr>
<tr>
<td>GCTs</td>
<td>Gate commutated thyristors</td>
</tr>
<tr>
<td>SGCTs</td>
<td>Symmetrical gate commutated thyristors</td>
</tr>
<tr>
<td>GTO</td>
<td>Gate turn-off thyristor</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse width modulated</td>
</tr>
<tr>
<td>CSI</td>
<td>Current source inverter</td>
</tr>
<tr>
<td>IGCTs</td>
<td>Insulated gate commutated thyristors</td>
</tr>
<tr>
<td>EIGTs</td>
<td>Injection enhanced gate transistors</td>
</tr>
<tr>
<td>DFIG</td>
<td>Doubly-fed induction generator</td>
</tr>
<tr>
<td>Mvar</td>
<td>Mega Volt Ampere Reactive</td>
</tr>
<tr>
<td>KV</td>
<td>Kilo volt</td>
</tr>
<tr>
<td>KW</td>
<td>Kilo watt</td>
</tr>
<tr>
<td>MW</td>
<td>Mega watt</td>
</tr>
<tr>
<td>Hz</td>
<td>Hertz</td>
</tr>
<tr>
<td>RLC</td>
<td>Resistive, Inductive, Capacitive</td>
</tr>
<tr>
<td>ADC</td>
<td>Analog digital conversion</td>
</tr>
<tr>
<td>DAC</td>
<td>Digital analog conversion</td>
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Chapter 1

INTRODUCTION

1.1 Overview

Recently, there is an increasing concern about the environment. The need to generate pollution-free energy has triggered considerable effort toward renewable energy (RE). RE sources such as solar, wind, flowing water and biomass offer the promise of clean and abundant energy. They do not generate any greenhouse gases and are inexhaustible. Wind Power, in particular, is especially attractive for wind potential countries like Denmark, Germany, USA, Sweden e.t.c. In the era of renewable energy, wind power is mostly developed and matured technology around the globe. It is converted into a more convenient alternating current (AC) power through an inverter system.

Power quality (PQ) problems of energy distribution is not new, But until recently, the impact of these problems have become public awareness. Advance in technology of semiconductor devices has led to a revolution in electronic technology over the past decade and have more tendency in future. However rise in the PQ problem is due to power equipments which include adjustable-speed motor drives (ASDs), electronic power supplies, direct current (DC) motor drives, battery chargers, electronic ballasts [1]. The distortion in the current is due to nonlinearity of the resistor. These nonlinear loads are constructed by nonlinear devices, in which the current is not proportional to the applied voltage. A simple circuit as shown in Figure 1.1 illustrates the concept of current distortion. In this case, a sinusoidal voltage is applied to a simple nonlinear resistor in which the voltage and current vary according to the curve shown. While the voltage is perfectly sinusoidal, the resulting current is distorted [2]-[4].

![Figure 1.1](image)

Figure 1.1 Current distortion caused by nonlinear resistance [1]

Nonlinear loads seem to be the main source of harmonic distortion in a power distribution system. Non-linear load produce the harmonic currents and injected back to the power
distribution network by the point of common coupling (CCP). These harmonics current can interact negatively with a broad range of power systems equipment, especially capacity, transformer and motors and produce more losses, overheating and overloading.

There are set of conventional solutions to the harmonic distortion problems which have existed for a long time. The passive filtering is the simplest conventional solution to mitigate the harmonic distortion [5]-[6]. They are known as passive filters, because they do not depend upon an external power supply and/or they do not contain active component such as transistors. Although simple, these conventional solutions that use passive elements do not always respond correctly to the dynamics of the power distribution systems [7]. Over the years, these passive filters have developed to high level of sophistication. Some even tuned to bypass specific harmonic frequencies. However, the use of passive elements at high power level makes the filter heavy and bulky. Moreover, the passive filters are known to cause resonance, thus affecting the stability of the power distribution systems [9]. As the regulatory requirements become more stringent, the passive filters might not be able to meet future revisions of a particular Standard [9].

Remarkable progress in power electronics had spurred interest in active power filter (APF) for harmonic distortion mitigation. The active filters are made of passive and active components and require an external power source. The basic principle of APF is to utilize power electronics technologies to produce currents components that cancel the harmonic currents from the nonlinear loads [10]. Previously, majority of controllers developed for APF are based on analogue circuits [11]. As a result, the APF is inherently subjected to signal drift (See page.13). Moreover, the utilization of fast switching transistors (i.e. IGBT) in APF application causes switching frequency (see page.13) noise to appear in the compensated source current. This switching frequency noise requires additional filtering to prevent interference with other sensitive equipments [11]. The idea of hybrid APF has been proposed by several researchers [12].

A low cost passive high-pass filter (HPF) is used in addition to the conventional APF. The harmonics filtering task is divided between the two filters. The APF cancels the lower order harmonics, while the HPF filters the higher order harmonics. The main objective of hybrid APF is to improve the filtering performance of high-order harmonics while providing a cost-effective low order harmonics mitigation [13].

Digital controller using digital signal processor (DSP) or microprocessor is preferable, primarily due to its flexibility and immunity to noise signals. However it is known that using digital filter like wave digital filter (WDF) or circulator-tree wave digital filter (CTWDF) noise will not propagate to the electrical grid. Although using WDF or CTWDF hardware of the filter has been minimized and cost is going to be reduced.
1.2 Objective of Research

The objective of the research is:

✓ To propose a new filter which is wave digital filter (WDF) or circulator tree wave digital filter (CTWDF) for renewable energy and to propose the simple architecture of that filter.
✓ To simplify the architecture of that filter with respect to hardware.
✓ To describe the contribution of noise into electrical grid by using WDF or CTWDF.

The first objective of this study to synthesis the different filter like butterworth, Chebychev and Cauer as a circulator-tree wave digital filter. To plot the different response of that filter like magnitude response, frequency response and impulse response and to compare with the reference analog filter.

The other aim of this study is to use the existing model of Wind Farms and high voltage direct current (HVDC) lines from MATLAB and apply the different filter like the single tuned filter, band pass filter and low pass filter and high pass filter and compare the result with each other.

1.3 Thesis Organization

This thesis consists of this introductory chapter and five other chapters arranged as follows:

Chapter 2 This chapter covers the relevant literature related to this project, some general definitions. In addition this chapter has a description of Discrete Fourier Transform (DFT) and Fast Fourier Transform (FFT), and also some basic knowledge of harmonics and creation of harmonics and effect of harmonics.

Chapter 3 This chapter describes the different filters with some MATLAB plots and give the description of pole and zero theory with respect to stable and unstable system. HVDC and Wind farms both models are describe with the help of MATLAB/SIMULINK and also illustrate the result of both models by using of different filters.

Chapter 4 Conclusion and decision will be described on the basis of simulation results.

Appendix A (Future Work) Scattering matrix theory and Wave digital filter is described with algorithms and architectures. Implementation of circulator-tree wave digital filter Matlab result is given in this chapter. Comparison of simulation result with reference filter result is also described.
2-A Theory Section

In this section basic terms has to define on those basis, different types filters has to be designed and plot the different spectrums of the filters in the application section (2-B).

2.1 Fundamental of Harmonics

The definition of the harmonics is described as harmonics as voltages or currents at frequencies that are multiples of the fundamental frequency. In most systems, the fundamental frequency is 50 Hz. Therefore, harmonic order is 100 Hz, 150 Hz, and 200 Hz and so on.

Figure 2.1 illustrates that any periodic, distorted waveform can be expressed as a sum of pure sinusoids. The sum of sinusoids is referred to as a Fourier series, named after the great mathematician who discovered the concept. The Fourier analysis permits a periodic distorted waveform to be decomposed into an infinite series containing DC component, fundamental component (50/60 Hz for power systems) and its integer multiples called the harmonic components. The harmonic number (h) usually specifies a harmonic component, which is the ratio of its frequency to the fundamental frequency [4].

![Figure 2.1 Fourier series representation of a distorted wave [4]](image-url)
The total harmonic distortion (THD) is the most common measurement indices of harmonic distortion [3],[4]. THD applies to both current and voltage and is defined as the root-mean-square (rms) value of harmonics divided by the rms value of the fundamental, and then multiplied by 100% as shown in the following equation:

$$THD = \sqrt{\sum_{h>1}^{max} \frac{M_h^2}{M_1}} \times 100$$

where $M_h$ is the rms value of harmonic component $h$ of the quantity $M$.

THD of current varies from a few percent to more than 100%. THD of voltage is usually less than 5%. Voltage THDs below 5% are widely considered to be acceptable, while values above 10% are definitely unacceptable and will cause problems for sensitive equipment and loads [4].

The biggest problem with harmonics is voltage waveform distortion. We can calculate a relation between the fundamental and distorted waveforms by finding the square root of the sum of the square of all harmonics generated by a single load, and then dividing this number by the nominal 50/60 Hz waveform value. We do this by a mathematical calculation know as a Fast Fourier Transform (FFT) Theorem. This calculation method determines the total Harmonics Distortion (THD) contained within a nonlinear current or voltage waveform [14].

### 2.2 Fourier Series

Fourier series are used in the analysis of periodic functions or periodic signals into the sum of oscillating function called sines and cosines. Many of the phenomena studied in engineering and science are periodic in nature eg. the current and voltage in an alternating current circuit. These periodic functions can be analyzed into their constituent components (fundamentals and harmonics) by a process called Fourier analysis. By definition, a periodic function, $f(t)$ is that where $f(t) = f(t+T)$. This function can be represented by a trigonometric series of elements consisting of a DC component and other elements with frequencies comprising the fundamental component and its integer multiple frequencies. This applies if the following so-called Dirichlet conditions are met: [15]

- If a discontinuous function, $f(t)$ has a finite number of discontinuities over the period $T$
- If $f(t)$ has a finite mean value over the period $T$
- If $f(t)$ has a finite number of positive and negative maximum values

The expression for the trigonometric series $f(t)$ is as follows:
Where $\omega_0 = 2\pi / T$

We can further simplify Equation (1), and we get

$$f(t) = c_0 + \sum_{n=1}^{\infty} c_n \sin \left(n \omega_0 t + \phi_n\right)$$

Where

$$c_0 = \frac{a_0}{2}, c_n = \sqrt{a_n^2 + b_n^2}, \text{ and } \phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

Equation 2.4 is known as a Fourier series and it describes a periodic function made up of the contribution of sinusoidal of different frequencies.

$(n \omega_0)$ $nth$ order harmonic of the periodic function

c_o magnitude of the DC component

c_n and $\phi_n$ magnitude and phase angle of the $nth$ harmonic component

The component with $n = 1$ is the fundamental component. Magnitude and phase angle of each harmonic determine the resultant waveform $f(t)$. Equation 2.3 can be represented in as complex form as:

$$f(t) = \sum_{n=1}^{\infty} c_n e^{j n \omega_0 t}$$

Where $n = 0, \pm 1, \pm 2, \ldots$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-j n \omega_0 t} dt$$

Generally, the frequencies of interest for harmonic analysis include up-to 40th or so harmonics.

The main source of harmonics in power systems is the static power converter. Under ideal operation conditions, harmonics generated by a $p$ pulse power converter are characterized by [15]:

$$ln = \frac{l}{n}, \text{and } n = pl \pm 1$$

Where $n$ stand for the characteristic harmonics of the load; $l = 1, 2, \ldots$; and $p$ is an integer multiple of six.

A bar plot of the amplitude of harmonics generated in six-pulse converter normalized as $c_l/c_1$ is called the harmonic spectrum, and it is shown in Figure 2.2.

**Orthogonal Function**

A set of function, $\Phi_n$, defined in $a \leq x \leq b$ is called orthogonal (or unitary, if complex) if it satisfies the following condition:
Figure 2.2 Example of harmonics spectrum [15]:

\[
\int_a^b \phi_i(x) \phi_j(x) * dx = K_i \delta_{ij}
\]

Where \( \delta_{ij} = 1 \) for \( i = j \), and = 0 for \( i \neq j \), and * is the complex conjugate.

**Fourier Coefficient**

Integrating Equation (1) and applying orthogonal function, we can obtain the Fourier coefficients as follows [15]:

\[
a_0 = \frac{2}{T} \int_{-T/2}^{T/2} f(t) dt
\]

\[
a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(n \omega_0 t) dt, \quad \text{and,}
\]

\[
b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin(n \omega_0 t) dt
\]

Where \( h = 1,2,3,\ldots,\infty \).

**Even Function**

A function \( f(t) \), is called an even function if it has a following property:

\[
f(t) = f(-t)
\]

Figure 2.3 Example of even function [15]

**Odd Function**

A function is called odd function if:

\[
f(-t) = -f(t)
\]
Figure 2.4 Example of odd function [15]

An even function is a symmetrical to the vertical axis at the origin and an odd function is asymmetrical to the vertical axis at the origin. A function with a period $T$, is half-wave symmetrical if it satisfies to the condition [15]:

$$f(t) = -f(t \pm (T/2)) \tag{2.13}$$

**Example of Calculating Harmonics Using Fourier Series**

Figure 2.5 Shows the square wave function [15]

Consider the periodic function of Figure 2.5, which can be expressed as follow

$$f(t) = \begin{cases} 0, & -T/2 < t < -T/4 \\ 4, & -T/4 < t < T/4 \\ 0, & T/4 < t < T/2 \end{cases} \tag{2.14}$$

For which we can calculate the Fourier coefficient using Equation 2.8 through Equation 2.10 as follows:

$$a_0 = \frac{2}{T} \left( \int_{-T/2}^{T/2} f(t) \, dt \right) = \frac{2}{T} \left( \int_{-T/4}^{T/4} 0 \, dt + \int_{-T/4}^{T/4} 4 \, dt + \int_{T/4}^{T/2} 0 \, dt \right) \tag{16}$$

$$= \frac{2}{T} \left[ 4(T/4 + T/4) \right] = 4$$

$$a_1 = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(\omega_0 t) \, dt$$

$$= \frac{2}{T} \left( \int_{-T/2}^{T/2} 0 \cos(\omega_0 t) \, dt + \int_{-T/4}^{T/4} 4 \cos(\omega_0 t) \, dt + \int_{T/4}^{T/2} 0 \cos(\omega_0 t) \, dt \right) \tag{17}$$
We equally find that:

\[ a_i = \begin{cases} 
0 & \text{if } i = \text{even} \\
-1^{(i-1)/2} \frac{8}{\pi} & \text{if } i = \text{odd} 
\end{cases} \quad b_i = 0 \quad 2.17 \]

Therefore, from Equation 2.2, the Fourier series of this waveform is as follows:

\[ f(t) = 2 + \frac{8}{\pi} \left( \cos \pi t - \frac{1}{3} \cos 3\pi t + \frac{1}{5} \cos 5\pi t - \cdots \right) \quad 2.18 \]

### 2.3 Discrete Fourier Transform

Discrete time and frequency representations are related by the discrete Fourier transform (DFT) pair used with digitized signals. The fast Fourier transform (FFT), is a very efficient numerical method for computing a discrete Fourier transform and is an extremely important factor in modern digital signal processing. For a discrete/sampled signal, the frequency spectrum can be obtained as follows:

\[ X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N} \quad 2.19 \]

where \( N \) is the number of samples over the period \( T \), \( x(n) \) is the amplitude at each sample and \( k = 0, 1, 2, \ldots, N-1 \).

Each frequency is also here separated by \( 1/T \), with the highest frequency component at \( k = N/2 \).

The highest frequency becomes: \( \frac{N}{2T} \)
Figure 2.6 Describe the signal / data transfer in time domain and frequency domain system

When a time domain signal is diluted with zeros, the frequency domain is duplicated. If the time domain signal is also shifted by one sample duration the dilution, the spectrum will additionally be multiplied by sinusoid [15].

2.4 Magnitude Function

Magnitude of the transfer function as the function of the frequency indicates how much is the certain frequency component is present in the transfer function. The frequency response $H(j\omega)$ is the complex function of $\omega$. For this reason it is interesting to study both the value of $H(j\omega)$ and the phase $\Phi(\omega)$. $H(j\omega)$ can be written as [16]:

\[
H(j\omega) = |H(j\omega)|e^{j\Phi(\omega)} \tag{2.20}
\]

\[
H(j\omega) = H_R(\omega) + jH_I(\omega) \tag{2.21}
\]

where $H_R(\omega)$ and $H_I(\omega)$ is the real (even) and imaginary (odd) function of $\omega$, respectively.

The magnitude function is defined:

\[
|H(j\omega)| \triangleq \sqrt{H_R^2(\omega) + H_I^2(\omega)} \tag{2.22}
\]

Magnitude function is used to plot the spectrum of different filters. Figures 2.9, 210, 2.11, 2.12 and 2.13 show the magnitude function of the different filters like Butterworth, Chebyshev and Cauer. Magnitude function can be expressed using the logarithmic scale:

\[
20\log(|H(j\omega)|) \text{ [dB]}
\]
2.5 Group Delay

The term group delay is used to refer to the average time delay imposed over the range of frequencies the filter is designed to pass through. In other words group delay can be defined as the negative rate of change of phase with frequency. Figures 2.9, 2.10, 2.11, 2.12 and 2.13 show the group delay of the different types of filters like Butterworth, Chebyshev and Cauer. The mathematical expression can be defined as

\[ \tau_g(\omega) = -\frac{\partial \phi(\omega)}{\partial \omega} \]  

2.23

The group delay is the even, rational function of \( \omega \). Applications which require a small variation in group delay are, e.g., video, EEG, EKG, FM (frequency modulation) signals, and digital transmission systems, where it is important that the waveform is retained [16].

2.6 Transfer Function

The System behavior is to describe i.e. properties which are in terms input and output signals. The ratio of output of the system to the input of the system in the Fourier domain considering its initial conditions to be zero is said to be transfer function. Without knowing the system (black box) we can calculate the transfer function of any system. When the transfer function operates on the input, the output is obtained. Numerator and denominator show the zeros and poles of transfer function of any system respectively. Stability and instability of the different filters are defined by poles and zeros is shown in figure 2.9, 2.10, 2.11, 2.12, .213. Many discrete-time and digital systems such as digital filters can be described by difference equations with constant coefficient [16]. The input-output relation for an Nth-order linear time-invariant systems (LTI) system can be described by:

\[ y(n) = \sum_{k=1}^{N} b_k y(n-k) + \sum_{k=0}^{M} a_k x(n-k) \]  

2.24

A behavioral description of an LTI system is the transfer function which can be obtained by applying the z-transform to both sides of Equation 2.20 [16] [20]. We get

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} a_k z^{-k}}{\sum_{k=1}^{N} b_k z^{-k}} \]  

2.25
The transfer function for a linear time invariant system (LTI) system is a rational function in $z$ and can therefore be described by a constant gain factor and the roots of the numerator and denominator polynomials. The roots of the numerator are called zeros, since no signal energy is transmitted to the output of the system for those values in the z-plane. The roots of the denominator are called poles [16]. For a causal, stable system, the poles are constrained to be inside the unit circle, i.e., in the stop-band of the filter, in order to increase the attenuation in the stop-band. Another common case occurs in all-pass filters where the zeros are placed outside the unit circle. Each zero has a corresponding pole mirrored in the unit circle, so that [16]:

$$Z_{\text{zero}} = \frac{1}{Z_{\text{pole}}}$$

### 2.7 Switching Frequency

Changes in frequency are phase continuous when they do not cause discontinuities in the phase (or amplitude) of the output signal. The first phase value after a frequency change is an increment of the last phase value before the change. Switching speed can be applied to both output amplitude and frequency and is the delay time required by the synthesizer to change between two frequencies or two power levels or both. The delay is a combination of the time required for the synthesizer's dedicated processor to react to a command, the switch/blanking time, the dwell time, and the settling time.

### 2.8 Signal Drift

The frequency of the signal must be accurately and adaptively tuned during changes (drift) in the center frequency of the signal.

### 2.9 Current Source Inverter

The current source inverter uses silicon-controlled rectifiers (SCRs), gate commutated thyristors (GCTs), or symmetrical gate commutated thyristors (SGCTs). This converter is known as an active rectifier or active front end (AFE) [21]. The DC link uses inductors to regulate current ripple and to store energy for the motor. The inverter section comprises gate turn-off thyristor (GTO) or symmetrical gate commutated thyristor (SGCT) semiconductor switches. These switches are turned on and off to create a pulse width modulated (PWM) output regulating the output frequency [21].
2.10 Voltage Source Inverter

The voltage source inverter topology uses a diode rectifier that converts utility/line AC voltage (50/60 Hz) to DC. The converter is not controlled through electronic firing like the current source inverter (CSI) drive [21]. The DC link is parallel capacitors, which regulate the DC bus voltage ripple and store energy for the system. The inverter is composed of insulated gate bipolar transistor (IGBT) semiconductor switches. There are other alternatives to the IGBT: insulated gate commutated thyristors (IGCTs) and injection enhanced gate transistors (IEGTs) [21].

2.11 Triplen Harmonics

Electronic equipment generates more than one harmonic frequency. For example, computers generate 3rd, 9th, and 15th harmonics. These are known as triplen harmonics. They pose a bigger problem to engineers and building designers because they do more than distort voltage waveforms. They can overheat the building wiring, because nuisance tripping, overheat transformer units, and cause random end-user equipment failure [14].

2.12 Circuit Overloading

Harmonics cause overloading of conductors and transformers and overheating of utilization equipment, such as motors. Triplen harmonics can especially cause overheating of neutral conductors on 3-phase, 4-wire systems. While the fundamental frequency and even harmonics cancel out in the neutral conductor, odd-order harmonics are additive. Even under balanced load conditions, neutral currents can reach magnitudes as high as 1.73 times the average phase current [14].

2.13 Sensitivity

Digital filters are not affected by temperature variations, variation in power supply, stray capacitances etc., which is an annoyance in analog filters. The reason for this is that the properties of a digital filter are determined by the numerical operations on numbers and not dependent on any tolerances of electrical components. For the same reason there is no aging or drift in digital filters. Independence of element sensitivity leads to high flexibility, miniaturization and high signal quality [16].
This additional loading creates more heat, which breaks down the insulation of the neutral conductor. In certain cases, it breaks down the insulation between windings of a transformer. In both cases, the result is a fire hazard. But, one can reduce this potential damage by using sound wiring practices [5]. To be on the safe side, more engineers are doubling the size of the neutral conductor from feeder circuits to panel boards and branch circuit partition wiring to handle the additive harmonic currents [5].

2-B Application Section

This section describes the different types of filters and the output septum of filters. These filters are used in existing wind farm models and HVDC lines to mitigate the harmonics, increase power quality and reduce losses of the electrical power system.

2.14 Analyzing Different Types of Filters

Below Figure 2.7 shows the subdivision of power filters.

Many filter solutions, called filter approximation, have been developed to meet different requirements, particularly for analog filters. The main work has focused on approximations to low-pass filters. Since high-pass, band-pass and stop-band filters can be obtained from low-pass filters through frequency transformations.

![Figure 2.7 Subdivision of active power filter][11][12]
It is also possible to use these results to design digital filters. The classical low-pass filter approximations, which can be designed by using most standard filter design programs are given below.

### 2.14.1 Butterworth Filter

The mathematically simplest and therefore most common approximation is Butterworth filters. Butterworth filters are used mainly because they are easy to synthesize and not because they have particularly good properties. The filter order must be an integer, and we therefore, but not always, select \( N \) to the nearest highest integer. The finite zeros of a transfer function are the roots of the numerator. Hence, Butterworth filters have no finite zeros since the numerator is a constant. The transfer function of Butterworth filters can be calculated by equations 2.24 and 2.25. The number of zeros is equal to the number of poles. Hence, Butterworth filters have \( N \) transmission zeros at \( s = \infty \) and \( N \) reflection zeros at \( s = 0 \). Butterworth filter has only poles and lacks finite zeros therefore all zeros lies at \( s = \infty \). Butterworth filters are therefore said to be of all-pole type.

The magnitude function is maximally flat at the origin and monotonically decreasing in both the pass-band and the stop-band as shown in figure 2.9. The variation of the group delay in the pass-band is comparatively large. However, the overall group delay is larger compared to the other filter approximations as shown in figure 2.9. The magnitude and group delay of the Butterworth filters can be calculated by equations 2.22 and 2.23 respectively. This approximation requires a larger filter order than the other filter approximations to meet a given magnitude specification.

---

**Figure 2.8** Basic kind of filter response [17]
2.14.2 Chebyshev Filter

Butterworth filter does not use the allowed pass-band tolerance efficiently. By allowing the magnitude function to vary within the acceptable pass-band bounds, a smaller transition band, than for a Butterworth filter of the same order, is obtained. For a Chebyshev I filter the magnitude function varies between the two tolerance bounds, that is, the filter has equiripple variation, i.e., the error oscillates with equal peaks across the magnitude response in the pass-band. An equiripple error is optimal in the Chebyshev sense. The transfer function is an all-pole function, i.e., all zeros are at infinity. The transfer function of Chebyshev filters can be calculated by equations 2.24 and 2.25.

The magnitude function has equal ripple in the pass-band and decreases monotonically in the stop-band and can be computed by equation 2.22. The variation of the group delay is somewhat worse than for the Butterworth approximation. The overall group delay is smaller than for Butterworth filters and can be computed by equation 2.23. A lower filter order is required compared to the Butterworth approximation.

---

**Figure 2.9** Output response of analog and digital Low-Pass Butterworth filter
2.14.3 Cauer Filter

The magnitude function has equal ripple in both the pass-band and the stop-band but the variation of the group delay is smaller than Chebyshev II. The Cauer filter, also called an elliptic filter, requires the smallest order to meet a given magnitude specification. These filter approximations represent extreme cases since only one property has been optimized at the expense of other properties. In practice they are often used directly, but they can serve as a starting point for an optimization procedure trying to find a solution that simultaneously satisfies several requirements. Cauer filter meets a standard magnitude specification with lower filter order than any other filter approximation. The transfer function of Cauer filter has finite zeros and can be calculated by equation 2.24 and 2.25. Filters of odd-order has a zero at $s = \infty$, but for filters of even-order the magnitude function approaches the stop-band attenuation, $A_{\text{min}}$. Note that one of the pole pairs lies close to the $j\omega$-axis and that the lower finite zero pair lies close to the stop-band edge.
Figure 2.11 Output response of analog Low-Pass, High-Pass analog and digital Cauer filter

Figure 2.12 Output response of analog Low-Pass, Band-Pass analog and digital Cauer filter
Figure 2.13 Output response of analog Low-Pass, Band-Stop analog and digital Cauer filter
Chapter 3

SIMULATION RESULTS

3.1 Wind Farm Model Description

A 9 MW wind farm consisting of six 1.5 MW wind turbines connected to a 25 kV distribution system exports power to a 120 kV grid through a 30 km, 25 kV feeder. A 500 kW resistive load and a 0.9 Mvar (Q=50) filter are connected at the 575 V generation bus. Wind turbines using a doubly-fed induction generator (DFIG) consist of a wound rotor induction generator and an AC/DC/AC IGBT-based PWM converter as shown in figure 3.1. The switching frequency is 1620 Hz. The stator winding is connected directly to the 60 Hz grid while the rotor is fed at variable frequency through the AC/DC/AC converter.

The detailed model (discrete) such as the one presented in this Figure 3.1. The detailed model includes detailed representation of power electronic IGBT converters. In order to achieve an acceptable accuracy with the 1620 Hz switching frequency used in this Figure 3.1, the model must be discretized at a relatively small time step (5 microseconds). This model is well suited for observing harmonics and control system dynamic performance over relatively short periods of times (typically hundreds of milliseconds to one second).

Figure 3.1 shows the details model of wind farms with and without filter.

The DFIG technology allows extracting maximum energy from the wind for low wind speeds by optimizing the turbine speed, while minimizing mechanical stresses on the turbine during gusts of wind. The optimum turbine speed producing maximum mechanical energy for a
given wind speed is proportional to the wind speed. Simulation results will be taken before the 575V bus bar with and without the filter as mentioned in above figure 3.1.

The Single tune high pass filter is used to suppress the harmonics. In addition the Single tune filter is derived from Butterworth and Chebyshev I filters because these filters are based on poles and zeros. Butterworth and Chebyshev filters have no finite zeros and hence the numerator is constant. Single tune High-pass filters which are used for high-order harmonics and cover a wide range of frequencies. A special type of high-pass filter, the C-type high-pass filter, is used to provide reactive power and avoid parallel resonances. It also allows filtering low order harmonics (such as 3rd), while keeping zero losses at fundamental frequency.

![Figure 3.2](image1.png)

**Figure 3.2** shows the current and voltage wave simulation result without filter.

![Figure 3.3](image2.png)

**Figure 3.3** shows the voltage and current wave simulation result with filter.

As we clearly see the above simulation result that distortion level in the figure 3.2 is higher than the filter 3.3. The figure 3.3 simulation results shows the sinusoidal wave as compare to figure 3.2.

### 3.2 HVDC Line MATLAB Model With Three Phase RLC Shunt Filter

Three-phase harmonic filters are shunt elements that are used in power systems for decreasing voltage distortion and for power factor correction. Nonlinear elements such as power
electronic converters generate harmonic currents or harmonic voltages, which are injected into power system. The resulting distorted currents flowing through system impedance produce harmonic voltage distortion. Harmonic filters reduce distortion by diverting harmonic currents in low impedance paths. Harmonic filters are designed to be capacitive at fundamental frequency, so that they are also used for producing reactive power required by converters and for power factor correction.

The HVDC (High Voltage Direct Current) rectifier is built up from two 6-pulse thyristor bridges connected in series. The converter is connected to the system with a 1200-MVA Three-Phase transformer (three windings). A 1000-MW resistive load is connected to the DC side through a 0.5 H smoothing reactor. The filters set are made of the following four components of the powerlib/Elements library:

1. One capacitor banks (C1) of 150 Mvar modeled by a "Three-Phase Series RLC Load",

   Three filters modeled using the "Three-Phase Harmonics Filters" are used in HVDC line as shows in Figure 3.4.

   (1) One C-type high-pass filter tuned to the 3rd (F1) of 150 Mvar.

   (2) One double-tuned filter 11\textsuperscript{th}/13\textsuperscript{th} (F2) of 150 Mvar.

   (3) One high-pass filter tuned to the 24\textsuperscript{th} (F3) of 150 Mvar.

![Figure 3.4 HVDC line connected with RLC Three Phase Shunt Filter.](image)
In order to achieve an acceptable distortion, several banks of filters of different types are usually connected in parallel. The combination of different filters banks are derived from basic filters which are Butterworth, Chebyshev and Cauer filters described in Section 2-A (2-A Application Section). The most commonly used filter types are:

1. Band-pass filters, which are used to filter lowest order harmonics such as 5th, 7th, 11th, 13th, etc. Band-pass filters can be tuned at a single frequency (single-tuned filter) or at two frequencies (double-tuned filter).
2. High-pass filters, which are used to filter high-order harmonics and cover a wide range of frequencies. A special type of high-pass filter, the C-type high-pass filter, is used to provide reactive power and avoid parallel resonances. It also allows filtering low order harmonics (such as 3rd), while keeping zero losses at fundamental frequency.

![Diagram of filter types](image)

**Figure 3.5** shows the different types of three-phase RLC harmonic filter.

Different simulations were done with the help of RLC three-phase harmonic filters for HVDC line, to reduce the voltage and current distortion in the power system and also increase the power factor. Given below simulation results three types of different filter combination were used for the HVDC lines:

1. 

![Simulation diagram](image)

**Figure 3.6** The two single tune with one high-pass filter used for HVDC lines.
Figure 3.7 shows the voltage wave form of above circuit with and without filter.

Figure 3.8 shows the current wave forms of above circuit with and without filter.

2.

Figure 3.9 The two high-pass filter with one single tune filter used for HVDC lines.
Figure 3.10 Simulation result of voltage wave form with and without filter of circuit 3.9 for HDVC lines.

Figure 3.11 Simulation result of current wave form with and without filter of circuit 3.9 for HDVC lines.

Figure 3.12 The One double tune frequency filter with on high pass filter for HVDC lines.
After comparing the different simulation results, we conclude that the circuit with the figure 3.12 is much more better than the remaining circuit. Because in that given circuit there is one high pass filter, which remove the high order harmonics and double tune filter, which is dealing with two frequencies component and to remove the harmonics with two different frequency ranges.
Chapter 4

CONCLUSION & DISCUSSION

4.1 Conclusion

In this work, harmonics and power quality are analyzed. The modeling and simulations techniques of a wind power converter and connected power system had been analyzed using MATLAB and SIMULINK. The existing wind farms in MATLAB engineering simulation software are used to suppress the harmonics generated by electrical converter. In the existing wind farms model the single tune high-pass filter are used to suppress the high frequency harmonics component. The fundamental frequency of the system which is 50/60 Hz and the harmonics are generated with the multiple of fundamental frequency. The amplitude of the harmonics is smaller than the fundamental frequency but the harmonics frequency is greater than the fundamental frequency. Therefore result shows that high-pass filter is used to suppress the high frequency component.

In addition different types of three phase filters are used in the HVDC line connected in parallel, in order to achieve an acceptable distortion. The HVDC model in MATLAB/ SIMULINK is used with different three phase filter banks to reduce the distortion and to increase the power quality of the system. Three-phase harmonic filters are shunt elements that are used in power systems for decreasing voltage distortion and for power factor correction. After comparing the different three phase filters simulation results we found that the high pass with double tune filter is used mostly to achieve the acceptable distortion level. The results show that this filter has to remove the high frequency harmonics component as well as to remove the low frequency component. From the above result we conclude that above filter can be used to suppress to harmonics at different frequency level from 13th order to 23rd order.

There will be an increasing economical impact on the operation of electrical power system due to losses in the system. Hence the losses in the system are generated by distortions or harmonics in the electrical power system. Economical impact of the electrical power system can be decreased by mitigating or reducing harmonics and distortions and to increase the power quality of the system.
A-1 Introduction

A method to obtain a low-sensitive digital filter structure is to simulate a low-sensitive analog filter such that the sensitivity properties are retained. Insertion loss method can be used to design analog filters that have minimal element sensitivity. The simulated analog filter is called reference filter. An important property which wave digital filters inherit from reference filter is the guaranteed stability [16]. The inductors in an LC ladder filter are nonlinear. These nonlinearities can produce parasitic oscillations. The passive LC filter dissipates signal power so the parasitic oscillations are attenuated in these filters and eventually disappear completely. Wave digital filters are modular and possess a high degree of parallelism. Thus, they are easy to implement in hardware. Wave digital filters are suitable for high-speed applications [16].

\[
A_k = V_k + R_i I_k \\
B_k = V_k - R_i I_k \\
A_0 = \sum_{k=1}^{N} A_k
\]

Figure A.1a shows the two port adaptor [16].

Figure A.1b shows internal architecture of symmetric two-port adaptor.

Below the architecture and block diagram is described with the help of mathematic equation [16].
To find value of coefficient the below mathematical expression is used:

$$\alpha_N = 2 - \sum_{k=1}^{N-1} \alpha_k$$

Some basic transformation are used to describe the wave digital filter:

1.) **Feldtkeller's Equation**: Explains the low-sensitivity property of doubly resistively terminated LC filter inherited by wave digital filters.

$$|H(e^{j\omega f})|^2 + |H_e(e^{j\omega f})|^2 = 1$$

2.) **Richards’ Transformation**: Mapping from $\varphi$ domain to $s$-domain.

3.) **Element Transformation**: Transformation of $S$, $\varphi$ and $Z$ domain with basic element components resistor, capacitor and inductor.

**Figure A.2a** Resistance and related wave flow diagram [18].

**Figure A.2b** Capacitance and related wave flow diagram [18].
Figure A.2c Inductance and related wave flow diagram [18].

A-2 Interconnection of Elements

Figure A.3 Two port adaptor in s-domain described the transmission and reflection [20].

By using the definition for voltage waves, the waves at the two ports is given according to

\[
A_k = V_k + R_k I_k \quad \text{A.7a}
\]

\[
B_k = V_k - R_k I_k \quad \text{A.7b}
\]

Where \( k = 1,2 \ldots \)

Kirchoff’s voltages- and currents laws gives the relationships according to

\[
V_1 = V_2
\]

\[
I_1 = I_2
\]

Eliminating voltages and currents from equation (4.7a, 4.7b), the relationship between the incident and reflected waves for the adaptor is:

\[
B_1 = A_2 + \alpha (A_2 - A_1) \quad \text{A.8a}
\]

\[
B_2 = A_1 + \alpha (A_2 - A_1) \quad \text{A.8b}
\]

The wave-flow graph for equation (A.8a, A.8b) is shown in Figure (A.4a, A.4b).
For $R_1 = R_2$ corresponding to $\alpha = 0$, which means no reflection. For $R_2 = 0$ corresponding to $\alpha = 1$ and the incident wave at port 1 is reflected and is phase-shifted by $180^\circ$ and $R_2 = \infty$ corresponding to $\alpha = 1$ which means incident wave is reflected by no phase-shift. The phase-shift of $180^\circ$ corresponds to a multiplication by -1[20].

![Figure A.4a](image)

**Figure A.4a** Wave flow of symmetric two port adaptor [20].

![Figure A.4b](image)

**Figure A.4b** Symmetric two port adaptor [20].

The Scattering Matrix which is described in equation 4.9:

$$A.9$$

By elimination of voltages and currents

$$A.10$$

and

$$A.11$$

If $Z_1$ and $Z_2$ are pure reactances, then $S_1$ and $S_2$ is two all-pass-functions.

If the second voltage source is short-circuit and using the first one as source it will results in $A_2 = 0$ and the reflected waves is given by:
The transfer function $H$ and the corresponding complementary transfer function is then

$$H = \frac{B_2}{A_1} = S_{21} = S_{12} = 0.5(S_2 - S_1)$$

$$H_c = \frac{b_1}{A_1} = S_{11} = S_{22} = 0.5(S_2 + S_1)$$

By using Feldtkeller’s equation which states that input power is equal to output- and reflected power, the realization of the two transfer functions is given by:

$$|H|^2 + |H_c|^2 = 1$$

It can be shown that for a low-pass and high-pass filter the order must be odd [20].

### A-3 3rd-Order Low-Pass Circulator -Tree WDF Implementation.

To implement the 3rd order lowpass cauer filter the following specification can be used $A_{max} = 1$ dB, $A_{min} = 35$ dB, $\omega_c T = 0.12\pi$ rad, and $\omega_s T = 0.6\pi$ rad. Minimum order is $N_{min} = 2.2515$, and which is increased to $N = 3$, since as we mention above that only odd order are feasible. After computing the order we compute the poles and zeros. We get the following poles and zeros respectively, which are given below

<table>
<thead>
<tr>
<th>Poles</th>
<th>Zeros</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1 = 0.8600 \pm 0.3342i$</td>
<td>$Z_1 = -0.6740 - 0.7387i$</td>
</tr>
<tr>
<td>$P_2 = 0.8600 + 0.3342i$</td>
<td>$Z_2 = -0.6740 - 0.7387i$</td>
</tr>
<tr>
<td>$P_3 = 0.8138$</td>
<td>$Z_3 = -1.0000$</td>
</tr>
</tbody>
</table>

Table A.1 show the poles and zeros values of 3rd low pass filter.

From these poles, zeros and gain, we get the impulse response. From this impulse response we compute the transfer function. After computing the transfer function, we get the magnitude and attenuation. The impulse response and attenuation of digital cauer filter is shown in Figure A.5 and A.6. Note that to synthesis the circulator-tree wave digital filter, we can compare the result like impulse response, attenuation and transfer function of digital cauer filter with circulator-tree WDF (Wave Digital Filter).

We get the following adaptor co-efficient

$$\alpha_1 = 0.0931$$

$$\alpha_2 = 0.0743$$

$$\alpha_3 = 0.0656$$
To get the adaptor coefficients, note that now we have to design the 3rd-order low-pass circulator-tree WDF. The first order section is shunt circuit, and other section is series resonance circuit. Figure 4.7 shows the design of 3rd order lowpass circulator-tree WDF. If we cut this structure from the middle, it is a symmetric structure, so that we gave the input impulse at $a_1 = 1$ and $a_2 = 0$ and to check the output impulse response at $b_2$ and vice versa for other input. Now we can compare the results of digital cauer filter with circulator-tree WDF. It seems like the same, also we can compare the transfer function and impulse response of both filters. From these results we can say that we realize the circulator-tree WDF accurately. Figures 6.14 and 6.15 show the impulse and attenuation response of second order sections of lowpass circulator-tree WDF respectively.

**Figure A.5** Impulse response of 3rd order low pass filter.

**Figure A.6** Attenuation of 3rd order low pass filter.

Figures A.8 and A.9 show the impulse and attenuation response of second order sections of low-pass circulator-tree WDF respectively.
Figure A.7 Low-pass circulator –tree WDF of third order filter

Figure A.8 Impulse response of 3rd order low-pass circulator tree WDF.

Figure A.9 Attenuation response of 3rd order low pass circulator-tree WDF.
### A-4 Data Conversion

Data conversion is used to convert the analog data into digital form by ADC and then convert back the digital data into analog form by DAC. Figure 4.10 the analog and digitl conversion block.

![Data Conversion Diagram](image)

**Figure A.10** show the data conversion block diagram.

The input of analog filter which is AC sinusoidal wave form. By using ADC (Analog Digital Conversion) to convert this sinusoidal AC voltage/current wave form into digital data. Digital processing circuit is used to remove the harmonics/distortion from that input data and fed into the DAC (Digital Analog Conversion). DAC convert the digital data into analog form and after that analog reconstruction filter is used, to convert the analog data into data into original input form.

Below diagram explain the data conversion of series into parallel and vice versa.

![Series Parallel Conversion Diagram](image)

**Figure A.11** show the series parallel data conversion.
References


